Effects of instanton interactions on the phases of quark matter

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Chapter 7

Summary

There are strong indications that in heavy-ion collisions a new phase of matter is created, quark matter, which is a state of matter with deconfined quarks. Besides being created in heavy-ion collisions, it is also believed to have existed in early universe. Today it might exist in the interior of very dense neutron stars. In this thesis we have studied how quark matter is influenced by instantons. These nonperturbative effects are closely related to the QCD vacuum angle θ. Because of the existence of instantons observables can become θ-dependent. In Nature θ appears to be very close zero, an additional argument for this was presented in Chapter 4 of this thesis. In heavy-ion collisions θ may effectively become nonzero, at least that conclusion is drawn from an effective low-energy theory of the strong interaction. When θ is different from $0 \pmod \pi$, the theory is not invariant under CP.

As effects of nonzero θ and instantons cannot be seen in perturbation theory and nonzero θ is also currently impossible to simulate on the lattice, effective theories and models have to be used. The most obvious one would be chiral perturbation theory, but unfortunately it is only valid at very low energies and for the ground state, not for the metastable states with effectively nonzero θ. Therefore, the investigations presented in this work were done using model calculations, the Nambu–Jona-Lasinio (NJL) model and the linear sigma model coupled to quarks. 't Hooft has shown that instantons induce an extra interaction in effective models, the 't Hooft determinant interaction. We studied the effects of this interaction on the phase structure of two-flavor quark matter. We studied, among others, the role that instantons play on phases of the strong interaction that spontaneously violate CP invariance.

We started the thesis with a short introduction to QCD, instantons and the consequences of a nonzero θ-angle in Chapter 1. Using chiral perturbation theory and experimental results we presented the arguments that $\theta < 10^{-10}$ in Nature. Furthermore, it was argued that for a certain range of parameters, metastable phases may become possible. These phases could be relevant in heavy-ion collisions and maybe for the early universe.

We continued our introduction with Chapter 2, where several facets of the QCD phase diagram were discussed. We first presented a short discussion about phase transitions and a review about the best-known phase diagram of QCD, the $\mu_B - T$-phase diagram. Then we introduced the three physical systems where quark matter is thought to play a role, the early universe, heavy-ion collisions and dense neutron stars. Also we presented several ways of obtaining theoretical knowledge about the QCD phase diagram. Finally, we discussed some non-standard phase diagrams, the QCD phase diagram as a function of the current quark masses, the isospin chemical potential, and θ.

In Chapter 3 the NJL model was introduced, which is a quark model with four-quark interactions that is a good description of low-energy QCD. We started with some historical background. Then we discussed the vacuum structure, which induces a large effective mass of the quarks, usually referred to as the constituent quark mass. Also we introduced the bound states of the model, which can be interpreted as mesons. Finally some low-energy relations were derived that can be used to fit the parameters of the model to data.

In Chapter 4 we presented a detailed study of the chiral symmetry breaking aspects of the phase diagram of the two-flavor NJL model at $\theta = \pi$. We concentrated on the effects of instantons and the violation of CP invariance. This chapter is in essence an extension of Chapter 2. We started the chapter with discussing the full θ-dependence at zero temperature and chemical potential, and later we investigated the case $\theta = \pi$ in more detail. The latter case is special, as it allows for the spontaneous breaking of CP invariance. The occurrence of this spontaneous breaking depends, among others, on the strength of the determinant interaction. If this strength reaches a critical value, which depends on the values of the current quark masses, spontaneous breaking of CP invariance occurs.

When the phase diagram is considered as a function of the up and down current quark mass at $\theta = \pi$ and a large enough value for the determinant interaction strength, a region in the diagram exists that spontaneously breaks CP invariance. In the NJL model both a lower and an upper boundary are found, in contrast to Tytgat (2000), who studied two-flavor chiral perturbation theory and only found a lower boundary. If the temperature is increased, the region becomes smaller and eventually disappears. This behavior may indicate that the suggestions for metastable states with an effective nonzero θ may not hold in QCD. It remains to be seen if these conclusions persist beyond the mean-field approximation and for the three flavor case.

Apart from the current quark mass dependence, also the dependence on temperature, baryon chemical potential and isospin chemical potential were considered. We presented phase diagrams as a function of either one of these three variables on one axis together with the strength of the instanton interaction on the other. It was found that when baryon chemical potential and temperature is increased, the CP violation disappears as a second order phase transition. This disappearance indicates that the violation of CP invariance is inherently a low energy phenomenon.

Also the mesons are affected by a nonzero CP-violating condensate. The mass eigenstates of the mesons are no longer CP and parity eigenstates. The condensate induces mixing between the mesons, the pions mix with the $\boldsymbol{a}_0$-mesons and the σ meson mixes with the η-meson.

In the phase diagram as a function of isospin chemical potential and strength of the determinant interaction at $\theta = \pi$, a novel phase with a nonzero $a_0^{\pm}$-condensate appears. Furthermore, the usual condition for a charged pion condensate is altered when the strength of the instanton interaction is larger than its critical value.

As we said previously, the phase that violates CP invariance in the NJL model disappears as a second order transition as a function of temperature. This is in disagreement with the findings of Mizher and Fraga (2009), who calculated, among others, this transition in a related model, the linear sigma model coupled to quarks (LSM*q*). In the latter model a first order transition is found. In Chapter 5 we discussed the similarities and differences between the two models. It was shown how one obtains a linear sigma model when the NJL model is bosonized. The important difference between the two models is the way quarks are included in the model. In the case of the NJL model, the quarks are necessarily taken into account at all temperatures, as it is a quark model. However, in the case of the linear sigma model coupled to quarks, quarks are only taken into account at nonzero temperatures.

The analysis presented in Chapter 5 shows, using Landau-Ginzburg arguments, that the quarks at zero temperature introduce a logarithmic term, which is not included in the LSM*q* model. A similar logarithmic term is obtained from quarks at high temperatures, which exactly cancels the zero-temperature term. This cancellation takes place in the NJL model, but not in the LSM*q* model. We showed that it is exactly this term that causes the qualitative differences between the two models.

In Chapter 6 the competition between the instanton interaction and a strong magnetic field was studied at $\theta = 0$. This study could be relevant for describing non-central heavy-ion collisions and the interior of neutron stars. Charged particles in strong magnetic fields are subject to Landau quantization, the effect that the momentum perpendicular to the magnetic field becomes quantized. This quantization can affect the phase structure of the matter involved considerably.

Firstly, because the quarks do not have the same charge, they behave differently in a magnetic field. As a function of baryon chemical potential it becomes possible that the two quarks have rather different constituent quark masses, a form of spontaneous isospin violation. Such violation can for instance affect the masses and decay rates of the mesons. The magnetic field effect is opposed by the instanton interaction, which favors equal behavior for the quarks: the constituent masses of the quarks and their phase transitions become coupled. However, when the strength of the instanton interaction is not too large, it is still possible to have a relatively large difference in constituent mass for the two quarks. This possibility disappears as the strength of the interaction is increased.

In addition the phase structure includes metastable states for a range of chemical potentials and magnetic fields. These states differ in the number of filled Landau levels and the amount of chiral symmetry breaking, which will affect the mesons accordingly. Furthermore, they are almost degenerate with the ground state and can therefore not be discarded.

Finally, we showed how a strong magnetic field affects the high-temperature phase transition, relevant for heavy-ion collisions. In the linear sigma model coupled to quarks it has been found that a strong magnetic field turns the crossover at zero magnetic field into a (weak) first order transition. In the NJL model we found that the transition remains a crossover. This different is important, as a first order transition and a crossover have very distinct experimental signatures, as in the case of a first order transition latent heat is absorbed or released, which does not happen in a crossover.

From the investigations reported in this thesis it is clear that instantons can play an important role in determining low-energy phenomena of the strong interaction and can affect the properties of quark matter. Hence, more detailed studies beyond the ones presented here would be valuable. The work we presented was performed using two-flavor effective models. It would be very interesting to extend our work by including the strange quark. In the chiral limit at $\theta = 0$, the order of the high-temperature phase transition is then different (Pisarski and Wilczek, 1984). It would be interesting to see whether this also holds for the CP restoring phase transition. Also, it would be interesting to see what happens beyond the mean-field approximation and when color superconductivity is included.

Another continuation of this work would be to calculate the equation-of-state and obtain mass-radius relations for stars obeying these relations, like for example Menezes et al. (2009a,b). From the mass-radius relation we would then see how the instantons and magnetic fields affect quark matter in compact stars.